Mathematics II

029

20/07/2021 **8.30 AM-11.30 AM**

NESA NATIONAL EXAMINATION AND SCHOOL INSPECTION AUTHORITY

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2020-2021

SUBJECT: MATHEMATICS II

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY **(MCB)**
- MATHEMATICS -COMPUTER SCIENCE-ECONOMICS **(MCE)**
- MATHEMATICS-ECONOMICS-GEOGRAPHY **(MEG)**
- MATHEMATICS -PHYSICS-COMPUTER SCIENCE **(MPC)**
- MATHEMATICS-PHYSICS-GEOGRAPHY **(MPG)**
- PHYSICS-CHEMISTRY-MATHEMATICS **(PCM)**

DURATION: 3 HOURS

INSTRUCTIONS:

1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets if provided.
2) Do not open this question paper until you are told to do so.
3) This paper consists of **two** sections: **A** and **B**.
   Section A: Attempt **all** questions. **(55 marks)**
   Section B: Attempt **only three** questions. **(45 marks)**
4) **Geometrical instruments and silent non-programmable calculators may be used.**
5) Use only a **blue** or **black** pen.

SECTION A: ATTEMPT ALL QUESTIONS (55 marks)

1) Calculate the limit **(3 marks)**

$$\lim_{x\to 4}\frac{\sin(x-4)}{x-4}$$

2) Write the following number in polar form: $w=\sqrt{3}-i$ **(4 marks)**

3) Convert $2x-5x^3=1+xy$ into Polar coordinates **(3 marks)**

4) Solve $\ln(e^x)=\ln(e^3)+\ln(e^5)$ **(2 marks)**

5) Solve the equation $x^2 2^x-2^x=0$ **(3 marks)**

6) Find the particular solution of $e^{dy/dx}=x+1$, given that $x=0,\ y=3$ **(4 marks)**

6) Find the equation of the plane which passes through point A (2, -1, 3) and parallel to the plane (P) with equation $x-2y+3z=1$ **(4 marks)**

8) Solve in $\mathbb{C}$ the equation $z^2=1+i$ **(4 marks)**

9) Consider the parabola given by $y=x^2+\sqrt{36x^2}+5$

a) Find its vertex. **(3 marks)**

b) Find its axis of symmetry. **(2 marks)**

10) Evaluate the integral $\int_0^{\frac{\pi}{2}}\frac{dx}{1+\tan^3 x}$ **(4 marks)**

11) Find the area of the region bounded by the graph of function $y=\sqrt{x^4}+1$ and the function $y=x^3$ and the vertical line $x=-1$ and $x=1$ **(4 marks)**

12) If the principal money is \$100,000 the annual interest rate is 5% and the interest is compounded continuously. What will be the balance after 40 years? **(4 marks)**

13) Differentiate the function $f(x)=\ln(x+\cos x)$ **(3 marks)**

14) Linearize $\sin x \cos^2 x$ **(4 marks)**

15) The curve $y=x^2-1$ is rotated about the x −axis through 360⁰. Find the volume of the solid generated when the area contained between the curve and the x −axis is rotated about the x −axis by 360⁰. **(4 marks)**

SECTION B CHOOSE ANY THREE QUESTIONS (45marks)

16) a) Determine the integral $\int \tan^3 x dx$ **(7 marks)**

b) Find the solution of differential equation $y''+8y'+25y=0$ which satisfies the initial conditions $x_0=0$, $y_0=2$ and $y'_0=1$ **(8 marks)**

17) a) The population $P1$ and $P2$ of two cities are given by the following Equations:

$$P_1=10000e^{kt}$$

$$P_2=20000e^{0.01t}$$

Where k is a constant and t is the time in years with $t=0$ corresponding to the year 2000. Find the constant k so that the two populations are equal in the year 2040 and approximate your answer to 3 decimals. **(7 marks)**

b) Find the Cartesian equation of the plane α which passes through the Point $P=(2;-3;4)$ and perpendicular to the line defined by the points a = (1, 5, 7) and b = (-2, 2, 3). **(4 marks)**

c) For what value of λ are the vectors $\vec{i}+2\vec{j}-3\vec{k}$, $3\vec{i}+\lambda\vec{j}+\vec{k}$ and $\vec{i}+2\vec{j}+3\vec{k}$ coplanar? **(4 marks)**

18) Point A $(3, 2)$ and B $(5, 0)$ are in Cartesian plane $(0,\vec{i},\vec{j})$. (scale: 1 unit to present $1\ cm$) Represent the point in the same Cartesian plane and construct the triangle OAB. **(2 marks)**

a) Find the equations of straight lines OA and AB. **(5 marks)**

b) Calculate the area of triangle OAB using the equation in (b) and integral calculation (calculations area in cm^2). **(4 marks)**

c) Prove the results applying the area of a triangle. **(2 marks)**

d) Deduce average value of f for $[0;\ 5]$. **(2 marks)**

19) Suppose a study of speeding violations and use of car phones by some drivers produced the following data:

	Speeding violation in last year	No speeding violation in last year	Total
Car phone use	25	280	305
No car phone use	45	405	450
Total	70	685	755

Calculate the following probabilities using the table:

a) P (person is a car phone user) **(3 marks)**

b) P (person had no violation in last year) **(3 marks)**

c) P (person had no violation in last year AND was a car phone user) **(2 marks)**

d) P (person is a car phone user OR person had no violation in last year) **(3 marks)**

e) P (person is a car phone user GIVEN person had a violation in last year) **(2 marks)**

f) P (person had no violation last year GIVEN person was not a car phone user) **(2 marks)**

20) For all natural n, the numerical function F_n is defined by $F_n(x) = \frac{x^n}{1+x^2}$, $x \in \mathbb{R}$. Given that $I_n = \int_0^1 F_n(x)dx$,

a) Calculate I_1 **(4 marks)**

b) Calculate $I_1 + I_3$ and deduce I_3 **(4 marks)**

c) Show that, for all natural p, $I_{2p} + I_{2p+2} = \frac{1}{2p+1}$ **(3 marks)**

d) Calculate I_2, I_4, and I_6. **(4 marks)**

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