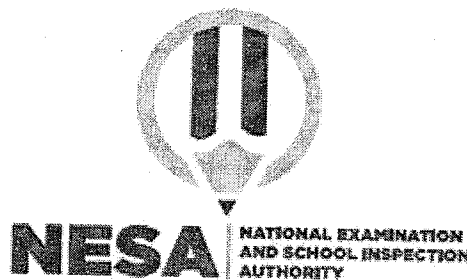


MATHEMATICS

MAT 02

28 /07/2022 08:30 AM – 11:30 AM



TTC NATIONAL EXAMINATIONS, 2021-2022

SUBJECT: MATHEMATICS

OPTION: SCIENCE AND MATHEMATICS EDUCATION (SME)

DURATION: 3 HOURS

INSTRUCTIONS:

- 1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets if provided.
- 2) Do not open this question paper until you are told to do so.
- 3) This paper consists of **two** sections: **A** and **B**.
Section A: Attempt **all** questions. (55 marks)
Section B: Attempt **any three** questions. (45 marks)
- 4) Geometrical instruments and silent non-programmable calculators may be used.
- 5) Show your working.
- 6) Use only a **blue** or **black** pen

SECTION A: ATTEMPT ALL QUESTIONS (55 marks)

- 1) Simplify $\frac{9^x \times 27^{x+1}}{81^{x-1}}$ (3 marks)
- 2) Solve the equation $1 + \frac{1}{x+2} - \frac{2x}{x^2-4} = 0$ (4 marks)
- 3) In how many different ways can 4 identical red balls, 3 identical green balls and 2 yellow balls be arranged in a row? (3 marks)
- 4) Find an equation of a straight line passing through the point (5, -6) and parallel to the straight line $3x - 2y + 10 = 0$ (3 marks)
- 5) Determine whether the functions $f(x) = \frac{2x+3}{x-1}$ and $g(x) = \frac{x+3}{x-2}$ are inverse functions by computing their compositions. (4 marks)
- 6) Find $\frac{dy}{dx}$ in term of x and y for $x^2 + xy = y^3$ (3 marks)
- 7) Find vector and parametric equations of the line L passing through points A and B such that $A(3, -2, 5)$ and $B(-1, 4, 2)$ (4 marks)
- 8) Calculate $\int \sin x \ln(1 + \sin x) dx$ (4 marks)
- 9) Form an arithmetic progression of three numbers such that the sum of its terms is 30 and the sum of squares of its terms is 332. (3 marks)
- 10) The difference of two numbers is 5 and their sum is 19. What are those numbers? (3 marks)
- 11) Solve this system of equations by using matrices. (5 marks)

$$\begin{cases} 2x + y - 3z = -4 \\ 4x - 2y + z = 9 \\ 3x + 5y - 2z = 5 \end{cases}$$

- 12) Calculate the $\lim_{x \rightarrow \infty} \sqrt{x^2 - 3x - 1} - x$. (4 marks)
- 13) Show that $\overline{P \wedge Q} \Leftrightarrow \overline{P} \vee \overline{Q}$ is a tautology. (4 marks)
- 14) Solve $\begin{cases} \log(x+y) = 1 \\ \log_2 x + 2 \log_4 y = 4 \end{cases}$ (4 marks)

- 15) Solve the differential equation to find its general solution $y'' + 2y' + y = 0$
(4 marks)

SECTION B: ATTEMPT ANY THREE QUESTIONS (45 marks)

- 16) Each student in a class of 32 students likes at least one of the following three subjects "Mathematics, Physics and Chemistry". It is found that 23 like Mathematics, 15 like Physics and 13 like Chemistry. 7 students like Mathematics and Physics, 9 like Mathematics and Chemistry and 6 like physics and Chemistry.
- Represent this information in a venn diagram and find the number of students who like all the three subjects. (5 marks)
 - How many students do like Mathematics only? (3 marks)
 - How many students do like Chemistry only? (3 marks)
 - Determine the total number of students who like at most one of the three subjects. (4 marks)
- 17) a) Given the circle equation $x^2 + y^2 - 4x - 2y = 4$
- Find the center (3 marks)
 - Find its radius (2 marks)

- b) Solve in \mathbb{R} the trigonometric equation:

$$\cos x + \sqrt{3} \sin x = \sqrt{3}$$

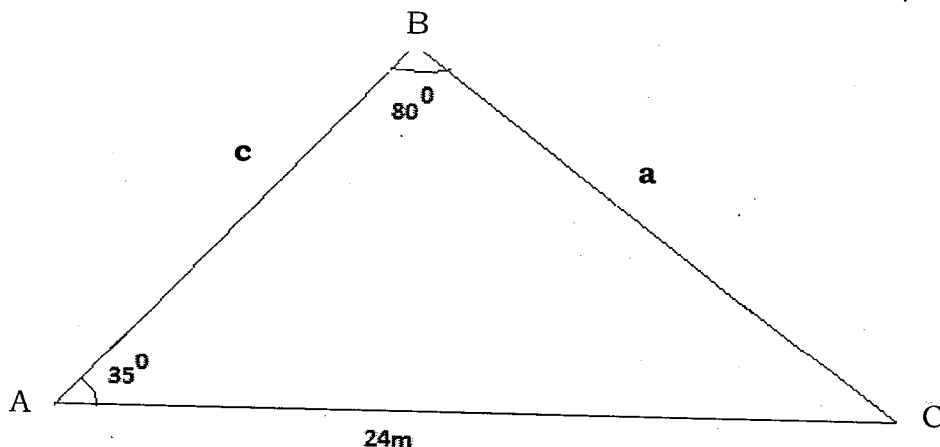
(5 marks)

- c) For what values of a is the matrix $A = \begin{bmatrix} 3-a & 8 \\ 1 & -4-a \end{bmatrix}$ singular?

(5 marks)

- 18) a) Determine the lengths of sides a and c in the triangle below.

(8 marks)



- b) Use Euler formula to show that $\sin x \cos^2 x = \frac{1}{4}(\sin 3x + \sin x)$.

(7 marks)

- 19) The data in the table below shows the heights of the members of a high school basketball team.

Height (cm)	177	178	179	181	183	184	186	189
Frequency	1	1	2	1	3	2	1	1

- a) Draw a distribution table of the data. **(10 marks)**
b) Find the mean of the distribution. **(3 marks)**
c) Calculate the standard deviation for the distribution. **(2 marks)**

20) Let the function defined by $f(x)=\ln(x^2+1)$

- a) Determine its domain of definition, study if the function is even, odd or if the function is periodic. **(2 marks)**
b) Calculate the limits and determine the asymptotes **(2 marks)**
c) Find the first derivative and its signs. Deduce the extremum if possible. **(2 marks)**
d) Find the second derivative and its sign. Deduce points of inflection if possible. **(2 marks)**
e) Establish the table of variation. **(1 mark)**
f) Determine the coordinates of the points on the curve where that curve cuts the axes (x-axis and y-axis). **(1 mark)**
g) Establish the table of values (supplementary points). **(2 marks)**
h) Sketch the curve of $f(x)$. **(3 marks)**

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